

ADA 039047

20000726075

NW

PREDICTING AFFECTED TEST OF ALTERNATIVE APPROACHES

Robert F. Lockman and John T. Warner

COM - Professional Paper-177

March 1977

Reproduced From
Best Available Copy

The ideas expressed in this paper are those of the author. The paper does not necessarily represent the views of either the Center for Naval Analyses or the Department of Defense.

CENTER FOR NAVAL ANALYSES

1401 Wilson Boulevard
Arlington, Virginia 22209

DDC
RECEIVED
MAY 5 1977
D

AD NO.

DDC FILE COPY

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

077 270

413

PREDICTING ATTRITION: ALTERNATIVE APPROACHES

- o BACKGROUND OF PREDICTING ATTRITION
- o ALTERNATIVE APPROACHES
- o TESTING THE APPROACHES
- o IMPLICATIONS FOR RECRUITING POLICY

attrition with varying degrees of success, but they must be specially administered to applicants if they are to be used for screening purposes.

Criticisms have been made of these past studies and current ones that employ personal characteristics and entry test scores to predict premature attrition (reference 4). The low value of the correlation of the predictors with the stay/attrite criterion has been cited, e.g., the R of .35 mentioned earlier. However this magnitude of correlation compares favorably with the validity coefficients of measures used to predict occupational performance in the civilian and military worlds (reference 5). It has been said that the low level of predictability is due to a decreasing diversity of the AVF manpower pool which limits the degree of correlation that can be achieved. But if this were true, the correlation could be corrected for such restriction without too much effort. The use of "static" personal characteristics and entry test scores has also been criticized because important "dynamic" situational or organizational variables are ignored. The desirability of investigating such measures for in-service classification and assignment purposes is evident (we ourselves are currently doing this for the Navy), but their reliability and validity for predicting attrition in conjunction with the "static" measures still must be demonstrated. Finally, it has also been said that the use of personal characteristics and entry test scores results in self-fulfilling prophecies of attrition - if men are thought to be dumb

and uneducated, they will be expected to fail and, therefore, will fail. There are compelling reasons for not labeling men with educational levels and mental groups, but at the same time our society places different values on these characteristics, and it is gratuitous to expect the services to do otherwise.

In any event, attrition, like death and taxes, is always with us, and today it is with us more than it was during the draft era. The three to four years premature loss rates in the 1960s ran from about 25 to 30 percent. Today, the comparable rates are 30 to 40 percent (references 1, 2, 3, and 6).

SLIDE 2

Costs of premature attrition are up, not only absolutely but relatively with the higher pay for today's volunteers and increased recruiting and training costs. The Navy estimates that it costs \$1,500 just to "access and dress" a non-prior-service recruit; another \$1,500 to get him (or her) through 8 weeks of recruit training; another \$400 for two weeks of apprentice training for those who do not go to Class A (technical training) schools; and about \$1,800 for technical training that averages 6 weeks (references 7 and 8).

These stages occur before a man is assigned to the fleet and becomes a productive member of the Navy. And as men are lost anywhere along the line, the toll mounts up. The costs of administrative and disciplinary discharges, unauthorized absences, desertion,

AVERAGE COSTS

ACCESS AND DRESS	\$1,500
8 WEEKS RECRUIT TRAINING	1,500
2 WEEKS APPRENTICE TRAINING	400
6 WEEKS TECHNICAL TRAINING	1,800

disciplinary measures, medical procedures, and the burden of dealing with unproductive losses-to-be also must be added to the bill.

In sum, then, premature losses, even of the voluntary type now undergoing experimental review in the Navy, are significant and expensive. Since personal characteristics and test scores are useful for screening out loss-prone applicants, the question is, what is the best approach for doing so?

ALTERNATIVE APPROACHES

When we talk about the "best" approach for screening out loss-prone applicants, we mean the most valid and least expensive, subject to the available supply of manpower. If the pool or potential recruits is so small that virtually all applicants have to be taken to meet manpower requirements, then screening is useful only for putting a "watch out" tag on a man whose chances of completing an initial tour are dim. If there is flexibility in whom we can take, screening becomes more useful in denying entry to the poorer risks.

There are two bases for screening. The first one is actuarial. With a sufficiently large recruit cohort, actual loss rates could be calculated for men with different patterns of characteristics. The trouble here, even when data is available on hundreds of thousands of men, is that we cannot be sure which are the most important characteristics, and combinations thereof, that relate to losses.

Statistical approaches to predicting attrition overcome the drawbacks of the actuarial approach. They let us know what the significant combinations of characteristics are that relate to losses and smooth out the projected rates.

SLIDE 3

There are two main but different statistical approaches that can be taken, with two variants of each. The main approaches are linear and non-linear in form, with the variants being the use of either individual or grouped observations.¹

The linear approach with individual observations is the most common. It was used in the early work of Flag, Caylor, and Flyer for the Navy, Army, and Air Force, respectively. Recently, it has been applied by the Navy Personnel R&D Center. The grouped linear and non-linear approaches are ones that I used recently for the Navy. The individual non-linear approach has been proposed by Dempsey and Fast to the Air Force.

Let us briefly look at the main features of these approaches and compare their pros and cons.

The linear approach with individual observations is the most familiar one. Numerous computer programs for regression analysis using this approach are available. These programs can easily handle

¹See the appendix for a technical discussion of these approaches.

COMPARISON OF APPROACHES

<u>Approach</u>	<u>Sample size</u>	<u>No. of variables</u>	<u>Computation</u>	<u>Data fit</u>
Linear - individual	Any	Many	1 stage	Poor
- grouped	Large	Fewer	2 stage	Fair
Non-linear - grouped	Large	Fewer	2 stage	Good
- individual	Large	Fewer	Iterative	Best

very large samples of men and many predictor variables in a one-stage analysis. The major disadvantage of the individual linear approach is that it may not be efficient, especially when the relationship of the predictors to the chances of attriting is not linear.

Whereas the individual linear approach uses a binary dependent variable, stay-attrite, the grouped approaches use loss rates (linear) or the log of the odds of loss rates (non-linear) for groups of men defined by all possible combinations of the predictors. An example of a group is recruits with 12 years of education, MG II, age 17, Caucasian, and no dependents. The groups are weighted to take account of their varying size in a regression analysis that is similar to the one performed with the individual linear approach.

Both grouped approaches require redefinition or pooling of groups and an additional regression when a predictor variable is found not to be significantly related to the dependent variable. Both also require very large samples with even small numbers of predictors. Because of the large number of possible combinations of the predictors, enough men must be found in the groups to produce reliable loss rates.¹

The grouped linear approach has the same major disadvantage as its individual counterpart when the relationships of predictors and loss rates is not linear. The grouped non-linear avoids this problem.

¹In our case, we have 3 levels of education, 5 of mental group, 3 of age, and 2 each of race and dependents. The product of these is 180, the number of possible groups.

All of the approaches so far rely on ordinary least squares regressions to solve their attrition equations, even the grouped non-linear approach. The non-linear individual approach is estimated by a different method, maximum likelihood. It can handle equations where the dependent variable is not a simple linear combination of the predictors (as can the grouped non-linear). However, in some cases, it may be the most time-consuming approach computationally. This is especially true when large numbers of variables and large samples are used, because of the iterative searching for the best fit to the data.

In this age of computers and ability to process massive amounts of data, the major question about the four approaches just described is, does it make any difference which one is used with the same data base?

We sought to answer this question by using the same set of predictors for 67,000 non-prior service males who joined the regular Navy in calendar 1973. The object was to predict the attrition experience for these recruits after each one had had the opportunity to be in the Navy for one year.

SLIDE 4

PREDICTORS

LT12ED	-	less than high school graduation
*12ED	-	high school graduation
GT12ED	-	more than high school graduation
MGI	-	mental group AFQT percentiles 93 and above
MGII	-	mental group AFQT percentiles 65 to 92
*MGIIIU	-	mental group AFQT percentiles 49 to 64
MGIIIL	-	mental group AFQT percentiles 31 to 48
MGIV	-	mental group AFQT percentiles 30 and below
AGE17	-	17 years old
*AGE18-19	-	ages 18 and 19
AGE20+	-	age 20 or older
*CAUC	-	Caucasians
NON-CAUC	-	Non-Caucasians
PDEPS	-	primary dependents (wife, children)
*NDEPS	-	no primary dependents

The predictors were all dichotomous or binary variables used to maintain consistency with current Navy selection procedures. They are shown on the slide.

RESULTS

We separated the CY 1973 Navy enlisted cohort into two samples by alternately assigning the individuals in the data file to validation and cross-validation samples, respectively. The 2 samples were virtually identical in terms of their characteristics and average first-year attrition rate, which was about 17.5 percent. Then, each of the four approaches or models was estimated with the validation sample, producing four fitted equations.¹ Each of these equations contained the same independent variables or predictors previously mentioned.

We then determined how well each equation predicted the attrition in the cross-validation sample. Our procedure for judging the "goodness of fit" was as follows. First, we used each fitted equation to predict the probability that each individual in the cross-validation sample would be a "stayer" rather than an "attriter" (which is one minus the individual's predicted attrition probability.) The Navy calls the probability of staying the individual's SCREEN score. SCREEN stands for Success Chances

¹The parameter estimates for the different models are shown in appendix B.

for REcruits Entering the Navy. Then we picked a critical SCREEN cut score, the score that separates people who will be accepted from those who will be rejected, and looked at the pattern of results.

SLIDE 5

We looked at:

- (1) How many of the predicted stayers actually stayed,
- (2) How many of the predicted attriters actually attrited,
- (3) How many of the predicted stayers actually attrited,
and, finally,
- (4) How many of the predicted attriters actually stayed.

The sum of (1) and (2) is the number of correct predictions, or "hits." Those who were predicted to stay but who attrite are called "false positives," and those who were predicted to attrite but actually stay are called "false negatives." Note that the percentage of false positives is the attrition rate the services would experience if they only took applicants with a SCREEN score above the cut score.

The success of each approach is judged by the percentages of hits, false positive and false negative predictions. As we will see, there is a tradeoff in identifying false positives and false negatives; you can reduce the percentage of false negative predictions only by increasing the percentage of false positive predictions. The "goodness" of a particular approach should be judged according to which percentage you are attempting to minimize, as well as by the percentage of hits.

PATTERNS OF RESULTS

<u>Predicted</u>	<u>Actual</u>	<u>Result</u>
Stay	Stay	Hits
Attrite	Attrite	
Stay	Attrite	False +
Attrite	Stay	False -

We looked at three different cut scores in comparing the alternative approaches. We will also see that the performance of the different approaches is crucially dependent upon the cut score chosen. The first cut score is 80, which was the mid-point between the average screen score of the actual stayers and the average score of the actual attriters. This score was chosen because this mid-point is conventionally used for classification purposes. The second cut score is 71, which was chosen because it is the Navy's current cut score. The third cut score is 76, which was selected because the Navy is considering raising the score to 76. In our comparisons, individuals with cut scores of 80 and below, 76 and below, or 71 and below, respectively, will be labeled attriters, and those with higher scores will be labeled stayers.

SLIDE 6

Now let us look at specific results. Here are the percentages of the sample that would be labeled attriters and therefore rejected under the alternative approaches and cut scores. As you can see, if the cut score is 71, about the same percentage of the cohort would be labeled attriters and therefore rejected under all four approaches. However, when the cut score is raised to 76 or 80, some differences between approaches emerge. If cut scores are based on either of the two linear models, a higher percentage of individuals would be rejected than when they are based on either of the two non-linear models.¹

¹See figure B-1 in appendix B.

PERCENT OF COHORT REJECTED AT VARIOUS CUT SCORES
UNDER DIFFERENT APPROACHES

<u>Prediction</u>	<u>Cut Score</u>	
	<u>71</u>	<u>80</u>
Individual linear	14	39
Grouped linear	15	37
Grouped non-linear	15	35
Individual non-linear	14	34

SLIDE 7

Let us now examine the percentage of hits, false positives, and false negatives obtained with each approach. Look first at the results for a cut score of 71. For this cut score, the percentage of hits, false positives, and false negatives are about the same for all four approaches. For the higher cut scores, however, the non-linear models outperform the linear ones in terms of hits and false negatives. The percentage of hits is higher for the non-linear approaches. The difference in hits between the linear and non-linear approach is most pronounced when the cut score is 80. The percentage of false negatives is slightly lower at a cut score of 76, but considerably at a score of 80. Remember that false negatives are those individuals predicted to attrite who actually stay.

Let's now look at the false positives. The percentage of false positives is the attrition rate that would actually be experienced. It is clear that higher cut scores lead to lower attrition rates. Now, it does appear that, at given cut scores, there would be more attrition when a screen table based on the non-linear approaches is used. There is a reason for the higher attrition under the non-linear approaches: they admit more people than the linear approaches, as we saw a few moments ago. The additional recruits admitted have somewhat higher attrition chances than the group already taken, and this raises the attrition rate of the selected cohort. However, this increase in attrition rates is small relative to the increased percentage of applicants

PERCENTAGE OF CORRECT, FALSE NEGATIVE, AND FALSE POSITIVE PREDICTIONS

<u>Prediction</u>	<u>Hits</u>		<u>False negatives</u>		<u>False positives</u>	
	<u>71</u>	<u>76</u> <u>80</u>	<u>71</u>	<u>76</u> <u>80</u>	<u>71</u>	<u>76</u> <u>80</u>
Individual linear	78	73 65	13	10 7	9	17 28
Grouped linear	78	73 66	13	10 8	9	17 27
Grouped non-linear	77	74 67	13	11 8	10	16 25
Individual non-linear	78	74 68	13	11 8	9	16 25

accepted and the decreased percentage of false negative predictions using the non-linear approaches

SLIDE 8

Our conclusions are shown on the next slide. If the cut score is 71, the score currently used by the Navy for general recruiting purposes, all four approaches will admit about the same number of recruits from any given cohort. Further, all four approaches produce about the same percentages of correct predictions ("hits"), false positives (predicted stays who attrite), and false negatives (predicted attrites who stay). At higher cut scores, the non-linear approaches are slightly better than the linear ones in that they admit more people from any given cohort, while yielding at least as high a percentage of correct predictions ("hits") and a lower percentage of false negatives (predicted attrites who stay). The non-linear approaches do, however, imply slightly higher actual attrition, since more people would be taken in using SCREEN tables based on these approaches.

The services are now under pressure from OSD and Congress to reduce first-term attrition, and one way to do this is to raise the cut score. As I mentioned earlier, the Navy is considering raising its cut score from 71 to 76. While the results with the alternative approaches at a cut score of 71 were not very different, they are at a cut score of 76. Of a cohort of 100,000 applicants, about 2,000 more would be screened out using one of the linear approaches rather than one of the non-linear

CONCLUSIONS

If cut score is 71:

- o All approaches admit about same number of recruits
- o All approaches have about same percentages of correct, false positive, and false negative predictions

With higher cut scores:

- o Non-linear approaches admit more people and consequently entail a higher attrition rate
- o Non-linear approaches yield a lower percentage of false negatives

approaches. Since the supply of manpower is limited and growing more so all the time, the services do not want to reject more applicants than is absolutely necessary to achieve some desired attrition rate. The more stringent the cut score, the better the non-linear approaches, since they do not unnecessarily screen out applicants and since they produce more hits, and fewer false negatives.

Let me close by noting one thing that remains to be done. This is to identify the optimal cut score. Raising the cut score is a way of reducing first-term attrition, but such a policy entails the cost of a reduced supply of acceptable manpower. This way of reducing attrition should be pursued only if the marginal costs of attrition exceed the costs imposed because end-strength goals are not met. Our future work will try to get at these costs and determine the optimal cut score.

REFERENCES

1. Plag, J.A. A Decade of Research in the Prediction of Naval Enlistee Effectiveness. U.S. Navy Neuropsychiatric Research Institute, San Diego, California, Report No. 70-21, 1971.
2. Caylor, J.S. Relationships between Army Recruit Characteristics and First Tour Performance. Human Resources Research Office, Washington, D.C., Technical Report 69-5, April 1969.
3. Flyer, E.S. Prediction of Unsuitability Among First-Term Airmen from Aptitude Indexes, High School Reference Data, and Basic Training Evaluations. Personnel Research Laboratory, Aerospace Medical Division, Air Force Systems Command, San Antonio, Texas, Technical Documentary Report PRL-TDR-63-17, June 1973.
4. Goodstadt, B.E. and Glickman, A.S. The Current Status of Enlisted Attrition in the U.S. Navy and U.S. Marine Corps and the Search for Remedies. American Institutes for Research, Washington, D.C., Final Report AIR-54500-11/75-FR, November 1975.
5. Ghiselli, E.E. The Measurement of Occupational Aptitude. University of California Press, Berkeley, 1955.
6. Manpower Research and Data Analysis Center, An Investigation of Premature Separations of First Term Enlisted Men Entering Active Duty since FY 1971. 28 August 1975.
7. Chief of Naval Operations Memo, Analysis of Proposed Management Actions for Voluntary/Administrative Separations, Ser 96/60167, 25 September 1975.
8. Interservice Training Review Organization, Training Management Report, Ser 992F/642936, 2 December 1976.
9. Lockman, R.F., Chances of Surviving the First Year of service. Center for Naval Analyses, Arlington, Virginia, Study 1068, November 1975.
10. Dempsey, J.R. and Fast, J.C., Predicting Attrition: An Empirical Study at the United States Air Force Academy, Air Force Personnel Center, Randolph Air Force Base, Texas March 1976.
11. Maddala, G.S., "Limited Dependent Variable Models." Mimeo-graphed, University of Florida (1976).
12. Zedlewski, E.W. Estimation and Inference in Binary Response Regressions. Center for Naval Analyses, Arlington, Virginia, Professional Paper No. 93, May 1972.

APPENDIX A
ALTERNATIVE MODELS FOR ESTIMATING
ATTRITION PROBABILITIES

Given the variables thought to influence attrition, the goal is to estimate the probability that an individual will attrite. Let $\underline{X} = (X_1, \dots, X_K)$ be the vector of variables (the characteristics of the individual, such as mental ability and educational level) thought to affect attrition. Then, with n observations on individuals who have been in military service, of which n_1 individuals were attriters and $n_2 = n - n_1$ individuals were non-attriters, we want to estimate an equation for the probability that an individual with a given set of characteristics (\underline{X} vector) will attrite. The estimated equation may then be used for prediction purposes. In this case, the dependent variable is binary and assumes a value of 1 if the individual attrites and 0 if he does not. Models that incorporate such dependent variables are called limited dependent variable models.

There are two classes of limited dependent variable models. One posits a linear cumulative distribution function; the other posits an S-shaped or sigmoid cumulative distribution. For the sake of exposition, we will refer to them as linear and non-linear models, respectively.

LINEAR MODELS

Linear models are estimated by ordinary least squares (OLS). The method is simply to estimate the following regression equation:

$$(1) \quad Y_i = \beta_0 + \beta_1 X_{1,i} + \dots + \beta_K X_{K,i} + \epsilon_i$$

The dependent variable in this regression, Y_i , depends upon whether the data is grouped or ungrouped.

Individual Linear Probability Model

If the linear model is based on the individual observations, the dependent variable is assigned the value 1 if the individual attrites and the value 0 if he does not. We call this the individual linear model. This model was used by Plag to estimate attrition probabilities from the Navy (reference 1).

The individual linear model is closely related to the linear discriminant function (LDF), first proposed by Fisher (reference 4) in 1936 as a means for identifying binary group membership on the basis of a linear combination ($\lambda_1 X_1 + \lambda_2 X_2 + \dots + \lambda_K X_K$) of known characteristics. It can be shown that the LDF "best" weights to place on the characteristics (the λ 's) are directly proportional to individual linear regression coefficients.¹ In our case, therefore, the discriminant function solution to separating applicants who belong to the population called attriters from the applicants who belong to the population called non-attriters would be based on a linear regression on a binary dependent variable.

¹See Maddala (reference 11). The factor of proportionality between discriminant function weights and OLS regression coefficients is the residual sum of squares from the OLS regression divided by $n-2$.

The individual linear model is appealing because of the computational ease of OLS and because OLS is capable of handling very large sample sizes. On the other hand, it has some shortcomings. The most frequently cited difficulties are that (1) the error term (ϵ_i in (1) above) is not normally distributed, (2) the error term does not have a constant variance, and (3) there is no restriction to predicting a probability between 0 and 1, although a prediction outside of this range is inadmissible. The first and third criticisms are not so serious,¹ but the second criticism implies that even within the class of linear models, the individual linear approach is not a fully efficient estimation procedure.²

¹The first difficulty implies that t tests for significance of regression coefficients are not exact tests. Maddala (reference 11) shows that, despite the binary form of the dependent variable in the linear probability model, the t tests for the regression coefficients are exact tests. The third cited difficulty is not really a problem either. The services would always take individuals with predicted attrition probabilities less than zero and screen out individuals with predicted probabilities exceeding unity. With large samples, predictions outside the limits of 0 and 1 will occur infrequently anyway.

²The error variance may be shown to be

$$\sum_i \beta_i X_i (1 - \sum_i \beta_i X_i)$$

and is a function of the values of X. Since the error term is not constant, the OLS estimates of the β 's are not the most efficient, i.e., minimum variance, linear estimates.

Grouped Linear Probability Model

An alternative to the linear probability model based on the individual observations is the grouped linear probability model. In this model, the individual observations are grouped into cells on the basis of combinations of the X's, and the dependent variable is the proportion $\hat{p}_i = \frac{a_i}{n_i}$ of the n_i individuals in the i th cell who were attriters. \hat{p}_i is an estimate of the true probability P that individuals with a given set of characteristics will attrite. The total number of cells is the product, over the number of variables, of the number of intervals for each variable. Thus, if there are 3 education categories (e.g., <12 years, 12 years, >12 years), 5 mental categories (I, II, IIIU, IIIL, IV and V), 3 age categories (<18, 18-19, >19), and 2 race groups (Caucasians and non-Caucasians), there would be 90 cells. To estimate the β 's, \hat{p}_i is regressed on categorical, or binary, variables representing the different levels of each independent variable.

In cells which contain small numbers of observations, \hat{p}_i may not be a good estimator of the true probability P_i . The variance of \hat{p}_i is $P_i(1-P_i)/n_i$ and is inversely related to n_i , the number of observations in the cell. Since \hat{p}_i does not have constant variance, neither does the error term in the regression, and the regression estimates of the β 's are not minimum variance estimates. This problem is handled by multiplying each \hat{p}_i by $\frac{1}{\sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{n_i}}} = \sqrt{\frac{n_i}{\hat{p}_i(1-\hat{p}_i)}}$.

In cells which contain more observations, \hat{P} is a lower variance estimate of the true attrition probability; hence, in the regression more weight is given to those cells which contain the largest numbers of observations.

Even if individual linear and grouped linear approaches were fully efficient linear estimation procedures, they have a potential shortcoming. A plot of $P_i = \sum_{j=1}^k \hat{\beta}_j X_{ij}$, where the $\hat{\beta}_j$'s are the estimated coefficients, yields a straight line, because the linear probability models have linear cumulative distribution functions. However, studies have found that the plot of the actual P's (the cell proportions in the grouped linear model) against $\sum_{j=1}^k \hat{\beta}_j X_{ij}$ frequently takes the form of an S-shaped curve, or sigmoid (reference 12). If the cumulative distribution is S-shaped rather than linear, the linear probability models may provide poor fits to the data. Models which imply S-shaped cumulative distributions, in which the probability of attriting is not a simple linear function of its predictors, may provide more accurate fits to the data.

NON-LINEAR MODELS

Probability distributions which have S-shaped cumulative distributions can be employed to estimate the β 's. The two most common ones are the logistic and normal distributions. In each of these distributions, the random variable Z is assumed to be a linear function of $X_1 \dots X_k$, that is, $Z = \sum_{j=0}^k \beta_j X_j$ (where $X_0 = 1$).

Individual Logistic Distribution

Since the logistic distribution has the form $P = \frac{e^{-Z}}{1+e^{-Z}}$, the function to be estimated is given in (2).

$$(2) \quad P = \frac{\exp\{-(\beta_0 + \beta_1 X_1 + \dots + \beta_K X_K)\}}{1 + \exp\{-(\beta_0 + \beta_1 X_1 + \dots + \beta_K X_K)\}} = \frac{1}{1 + \exp\{\beta_0 + \beta_1 X_1 + \dots + \beta_K X_K\}}.$$

Equation (2) is a non-linear equation which may be estimated by the method of maximum likelihood (ML). To estimate (2), the likelihood function L is formed, and that set of β 's which maximizes the value of L is found. Since individual observations are used, this model is called the individual logistic model. The likelihood function is:

$$(3) \quad L = \prod_{Y_i=1} \frac{1}{1 + \exp\{\sum \beta_i X_i\}} \prod_{Y_i=0} \frac{\exp\{\sum \beta_i X_i\}}{1 + \exp\{\sum \beta_i X_i\}}$$

Since (3) is not a simple linear expression, the β 's have to be estimated using non-linear techniques.

The other most frequently assumed probability distribution in maximum likelihood is a normal distribution with unit variance. In this case, the attrition probability is given in (4):

$$(4) \quad P = \int_{-\infty}^{\frac{-\sum \beta_i X_i}{\sigma}} \frac{\exp\{-\frac{1}{2}t^2\}}{\sqrt{2\pi}} dt$$

The likelihood function for the normal distribution is the following:

$$(5) \quad L = \prod_{Y_i=1} \left\{ P\left(-\frac{\sum \beta_i X_i}{\sigma}\right) \right\} \prod_{Y_i=0} \left\{ 1 - P\left(-\frac{\sum \beta_i X_i}{\sigma}\right) \right\}$$

Again, we find the β 's that maximize L , and this has to be done using iterative methods. This model is called the probit model.

Since the probit model is based on a normal distribution with unit variance, the parameters $\beta_1 \dots \beta_K$ are all scaled by a factor $1/\sigma$, where σ is the unknown standard deviation. σ is not separately estimable, and it is arbitrarily assumed to be unity. The probit model was used by Dempsey and Fast (reference 3) to estimate attrition probabilities from the Air Force Academy.

While the logit and probit models look different, their cumulative distributions are very similar. Suppose that Z_1 is a random variable distributed normally with unit variance and Z_2 is a random variable distributed logistically. It may be shown that Z_2 has variance $\frac{\pi^2}{3}$. Further, it may be shown that Z_2 divided by its standard deviation, $\frac{\pi}{\sqrt{3}}$, is distributed approximately normally with unit variance. Therefore, $Z_2 = \sum \beta_j X_j$ need only be multiplied by $\sqrt{3/\pi}$ to be comparable to $Z_1 = \sum \beta_i X_i$ obtained from the probit model. The estimates differ only by the scale factor $\frac{\sqrt{3}}{\pi}$. Therefore, ML logit is virtually identical to ML probit (and vice versa).

Grouped Logistic Model

With large amounts of data, the β 's in (2) can be estimated using linear regression. The probability function in (2) can be transformed into the following log-linear equation, which may be estimated with OLS:

$$(6) \quad \ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots + \beta_K X_K$$

The dependent variable here is the logarithm of the odds of being an attriter, estimated by grouping the data into cells, just as in the grouped linear, and then using $\ln(\hat{P}_i/1-\hat{P}_i)$ rather than \hat{P}_i as the dependent variable in the regression. The grouped linear regression procedure was utilized by Lockman (reference 2) to estimate attrition probabilities from the Navy.

The error term in the grouped logit regression is non-constant and has the variance $\frac{1}{n_i \hat{P}_i (1-\hat{P}_i)}$. Therefore, weighting by the inverse of its estimated standard deviation, $\sqrt{n_i \hat{P}_i (1-\hat{P}_i)}$, yields a model with a constant variance error term. Again, this procedure places the largest weights on those cells containing the largest number of observations.

APPENDIX B

THE EMPIRICAL EQUATIONS OBTAINED WITH ALTERNATIVE APPROACHES

Table B-1 contains the parameter estimates obtained by applying the alternative statistical procedures to the data. The parameter estimates in the first column labeled individual logit were obtained by the method of maximum likelihood. The parameter estimates in the other three columns were obtained by the method of ordinary least squares. The numbers in the first two columns are estimates of the β 's in the logit probability function $P = \frac{1}{1 + e^{-\sum \beta_j X_j}}$. The numbers in the last two columns may be interpreted as estimates of the β 's in the linear probability function $P = \sum \beta_j X_j$. The "t" values for the different variables are in parentheses. (The "t" values for the individual logit parameter estimates are asymptotic "t" values - see Zedlewski (reference 12)). A "t" value of 1.96 or greater indicates that the coefficient is significantly different from zero at the .05 level; a "t" value of 2.58 or greater indicates significance at the .01 level.

ESTIMATES OF PARAMETER VALUES

Variable	Individual logit	Grouped logit	Grouped linear	Individual linear
Ed < 12	-.6715 (21.23) ^a	-.6557 (14.42)	.1093 (14.14)	.1072 (19.05)
Ed > 12	.3493 (4.51)	.2835 (2.87)	-.0318 (3.79)	-.0341 (3.82)
Mental Group I	1.1789 (9.32)	1.0398 (6.00)	-.0839 (9.65)	-.0842 (7.33)
Mental Group II	.2012 (4.50)	.2017 (3.60)	-.0200 (3.09)	-.0210 (3.61)
Mental Group IIIL	-.3446 (7.71)	-.3415 (6.00)	.0523 (6.20)	.0534 (8.01)
Mental Group IV	-.5805 (12.98)	-.5712 (9.75)	.0972 (10.04)	.0988 (13.69)
Dependents	-.3489 (5.52)	-.4027 (5.21)	.0391 (3.61)	.0509 (5.57)
Age < 18	-.1450 (3.24)	-.1664 (3.14)	.0242 (2.56)	.0231 (3.38)
Age > 19	-.1848 (4.13)	-.1689 (3.24)	.0221 (3.51)	.0237 (4.16)
Race (Non-Caucasian)	.1359 (3.04)	.0805 (1.28)	-.0369 (4.15)	-.0246 (3.27)
Constant	1.9594 (61.96)	1.9503 (40.87)	.1179 (20.79)	.1192 (23.71)
N	30,000	137	137	30,000

^a"t" values are in parentheses.

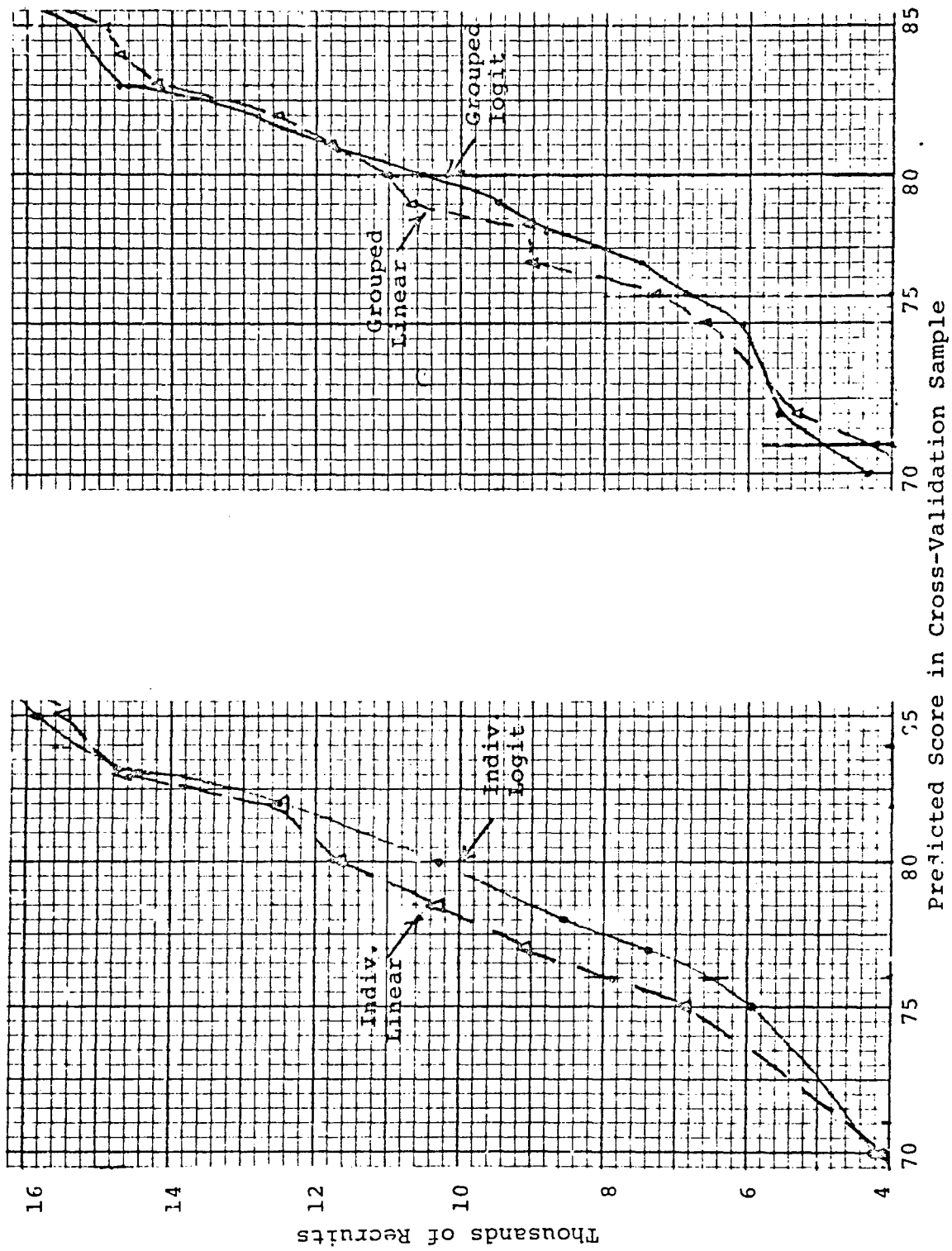


Figure B-1: Cumulative Predicted Score Distributions for the Four Approaches

CNA Professional Papers — 1973 to Present*

- PP 103
Friedheim, Robert L., "Political Aspects of Ocean Ecology" 48 pp., Feb 1973, published in *Who Protects the Oceans*, John Lawrence Hargrove (ed.) (St. Paul: West Publ'g. Co., 1974), published by the American Society of International Law AD 57 936
- PP 104
Schick, Jack M., "A Review of James Cable, Gunboat Diplomacy Political Applications of Limited Naval Forces," 5 pp., Feb 1973, (Reviewed in the American Political Science Review, Vol. LXVI, Dec 1972)
- PP 105
Corn, Robert J. and Phillips, Gary R., "On Optimal Correction of Gunfire Errors," 22 pp., Mar 1973, AD 761 674
- PP 106
Stoloff, Peter H., "User's Guide for Generalized Factor Analysis Program (FACTAN)," 35 pp., Feb 1973, (Includes an addendum published Aug 1974) AD 758 824
- PP 107
Stoloff, Peter H., "Relating Factor Analytically Derived Measures to Exogenous Variables," 17 pp., Mar 1973, AD 758 820
- PP 108
McConnell, James M. and Kelly, Anne M., "Superpower Naval Diplomacy in the Indo-Pakistani Crisis," 14 pp., 5 Feb 1973, (Published, with revisions, in *Survival*, Nov/Dec 1973) AD 761 675
- PP 109
Berghoefer, Fred G., "Salaries—A Framework for the Study of Trend," 8 pp., Dec 1973, (Published in *Review of Income and Wealth*, Series 18, No. 4, Dec 1972)
- PP 110
Augusta, Joseph, "A Critique of Cost Analysis," 9 pp., Jul 1973, AD 766 376
- PP 111
Herrick, Robert W., "The USSR's 'Blue Belt of Defense' Concept: A Unified Military Plan for Defense Against Seaborne Nuclear Attack by Strike Carriers and Polaris/Poseidon SSBNs," 18 pp., May 1973, AD 766 375
- PP 112
Ginsberg, Lawrence H., "ELF Atmosphere Noise Level Statistics for Project SANGUINE," 29 pp., Apr 1974, AD 786 969
- PP 113
Ginsberg, Lawrence H., "Propagation Anomalies During Project SANGUINE Experiments," 5 pp., Apr 1974, AD 786 968
- PP 114
Maloney, Arthur P., "Job Satisfaction and Job Turnover," 41 pp., Jul 1973, AD 763 410
- PP 115
Silverman, Lester P., "The Determinants of Emergency and Elective Admissions to Hospitals," 145 pp., 18 Jul 1973, AD 766 377
- PP 116
Rehm, Allan S., "An Assessment of Military Operations Research in the USSR," 19 pp., Sep 1973, (Reprinted from, *Proceedings, 30th Military Operations Research Symposium (U)*, Secret Dec 1972) AD 770 116
- PP 117
McWhite, Peter B. and Ratliff, H. Donald, "Defending a Logistics System Under Mining Attack," 24 pp., Aug 1976 (to be submitted for publication in *Naval Research Logistics Quarterly*), presented at 44th National Meeting, Operations Research Society of America, November 1973, AD A030 454
*University of Florida
**Research supported in part under Office of Naval Research Contract N00014-68-0273-0017
- PP 118
Barfoot, C. Bernard, "Markov Duels," 18 pp., Apr 1973, (Reprinted from *Operations Research*, Vol. 22, No. 2, Mar-Apr 1974)
- PP 119
Stoloff, Peter and Lockman, Robert F., "Development of Navy Human Relations Questionnaire," 2 pp., May 1974, (Published in *American Psychological Association Proceedings*, 81st Annual Convention, 1973) AD 779 240
- PP 120
Smith, Michael W. and Schnmper, Ronald A., "Economic Analysis of the Intracity Dispersion of Criminal Activity," 30 pp., Jun 1974, (Presented at the Econometric Society Meetings, 30 Dec 1973) AD 780 538
*Economics, North Carolina State University.
- PP 121
Devine, Eugene J., "Procurement and Retention of Navy Physicians," 21 pp., Jun 1974, (Presented at the 49th Annual Conference, Western Economic Association, Las Vegas, Nev., 10 Jun 1974) AD 780 539
- PP 122
Kelly, Anne M., "The Soviet Naval Presence During the Iraq-Kuwait Border Dispute: March-April 1973," 34 pp., Jun 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 780 592
- PP 123
Peterson, Charles C., "The Soviet Port-Clearing Operation in Bangladesh, March 1972-December 1973," 35 pp., Jun 1974, (Published in Michael McGwire, et al. (eds) *Soviet Naval Policy: Objectives and Constraints*, (New York: Praeger Publishers, 1974) AD 780 540
- PP 124
Friedheim, Robert L. and Jehn, Mary E., "Anticipating Soviet Behavior at the Third U.S. Law of the Sea Conference: USSR Positions and Dilemmas," 37 pp., 10 Apr 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 783 701
- PP 125
Weinland, Robert G., "Soviet Naval Operations—Ten Years of Change," 17 pp., Aug 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 783 962
- PP 126 — Classified.
- PP 127
Dragnich, George S., "The Soviet Union's Quest for Access to Naval Facilities in Egypt Prior to the June War of 1967," 64 pp., Jul 1974, AD 786 318
- PP 128
Stoloff, Peter and Lockman, Robert F., "Evaluation of Naval Officer Performance," 11 pp., (Presented at the 82nd Annual Convention of the American Psychological Association, 1974) Aug 1974, AD 784 012
- PP 129
Holen, Arlene and Horowitz, Stanley, "Partial Unemployment Insurance Benefits and the Extent of Partial Unemployment," 4 pp., Aug 1974, (Published in the *Journal of Human Resources*, Vol. IX, No. 3, Summer 1974) AD 784 010
- PP 130
Danilukes, Bradford, "Roles and Missions of Soviet Naval General Purpose Forces in Wartime: Pro-SSBN Operation," 20 pp., Aug 1974, AD 786 320
- PP 131
Weinland, Robert G., "Analysis of Gorkhov's *Navier in War and Peace*," 45 pp., Aug 1974, (Published in *Soviet Naval Policy*, ed. Michael McGwire; New York: Praeger) AD 786 319
- PP 132
Kleinman, Samuel D., "Racial Differences in Hours Worked in the Market: A Preliminary Report," 77 pp., Feb 1975, (Paper read on 26 Oct 1974 at Eastern Economic Association Convention in Albany, N.Y.) AD A 005 517
- PP 133
Squires, Michael L., "A Stochastic Model of Regime Change in Latin America," 42 pp., Feb 1975, AD A 007 912
- PP 134
Root, R. M. and Cuniff, P. F., "A Study of the Shock Spectrum of a Two-Degree-of-Freedom Nonlinear Vibratory System," 39 pp., Dec 1975, (Published in the condensed version of *The Journal of the Acoustic Society*, Vol 60, No. 6, Dec 1976, pp. 1314
*Department of Mechanical Engineering, University of Maryland.
- PP 135
Goudreau, Kenneth A.; Kuzmack, Richard A.; Wiedemann, Karen, "Analysis of Closure Alternatives for Naval Stations and Naval Air Stations," 47 pp., 3 Jun 1975 (Reprinted from "Hearing before the Subcommittee on Military Construction of the Committee on Armed Service," U.S. Senate, 93rd Congress, 1st Session, Part 2, 22 Jun 1973)
- PP 136
Stallings, William, "Cybernetics and Behavior Therapy," 13 pp., Jun 1975
- PP 137
Peterson, Charles C., "The Soviet Union and the Reopening of the Suez Canal: Mineclearing Operations in the Gulf of Suez," 30 pp., Aug 1975, AD A 015 376

*CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the author at the Center for Naval Analyses, 1401 Wilson Boulevard, Arlington, Virginia 22209.

CNA Professional Papers – 1973 to Present (Continued)

- PP 138
Stallings, William, "BRIDGE: An Interactive Dialogue-Generation Facility," 5 pp., Aug. 1975 (Reprinted from IEEE Transactions on Systems, Man, and Cybernetics, Vol. 5, No. 3, May 1975)
- PP 139
Morgan, William F., Jr., "Beyond Folklore and Fables in Forestry to Positive Economics," 14 pp., (Presented at Southern Economic Association Meetings November, 1974) Aug 1975, AD A 015 293
- PP 140
Mahoney, Robert and Druckman, Daniel, "Simulation, Experimentation, and Context," 36 pp., 1 Sep 1975, (Published in Simulation & Games, Vol. 6, No. 3, Sep 1975)
*Mathematica, Inc.
- PP 141
Mizrahi, Maurice M., "Generalized Hermite Polynomials," 5 pp., Feb 1976 (Reprinted from the Journal of Computational and Applied Mathematics, Vol. 1, No. 4 (1975), 273-277).
*Research supported by the National Science Foundation
- PP 142
Lockman, Robert F., Jehn, Christopher, and Shughart, William F. II, "Models for Estimating Premature Losses and Recruiting District Performance," 36 pp., Dec 1975 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A 020 443
- PP 143
Horowitz, Stanley and Sherman, Allan (LCdr., USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A021 581
- PP 144
Durch, William J., "The Navy of the Republic of China - History, Problems, and Prospects," 66 pp., Aug 1976 (To be published in "A Guide to Asiatic Fleets," ed. by Barry M. Blechman; Naval Institute Press) AD A030 460
- PP 145
Kelly, Anne M., "Port Visits and the "Internationalist Mission" of the Soviet Navy," 36 pp., Apr 1976 AD A023 436
- PP 146
Palmour, Vernon E., "Alternatives for Increasing Access to Scientific Journals," 6 pp., Apr 1975 (Presented at the 1975 IEEE Conference on Scientific Journals, Cherry Hill, N.C., Apr 28-30; published in IEEE Transactions on Professional Communication, Vol. PC-18, No. 3, Sep 1975) AD A021 798
- PP 147
Kessler, J. Christian, "Legal Issues in Protecting Offshore Structures," 33 pp., Jun 1976 (Prepared under task order N00014-68-A-0091-0023 for ONR) AD A028 389
- PP 148
McConnell, James M., "Military-Political Tasks of the Soviet Navy in War and Peace," 62 pp., Dec 1975 (Published in Soviet Oceans Development Study of Senate Commerce Committee October 1976) AD A022 590
- PP 149
Squires, Michael L., "Counterforce Effectiveness: A Comparison of the Tsipis "K" Measure and a Computer Simulation," 24 pp., Mar 1976 (Presented at the International Study Association Meetings, 27 Feb 1976) AD A022 591
- PP 150
Kelly, Anne M. and Petersen, Charles, "Recent Changes in Soviet Naval Policy: Prospects for Arms Limitations in the Mediterranean and Indian Ocean," 28 pp., Apr 1976, AD A 023 723
- PP 151
Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)
- PP 152
Mizrahi, Maurice M., "On Path Integral Solutions of the Schrodinger Equation, Without Limiting Procedure," 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-575).
*Research supported by the National Science Foundation
- PP 153
Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator," 137 pp., May 1976 (Submitted for publication in Annals of Physics) AD A025 440
*Research supported by the National Science Foundation
- PP 154
Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (To appear in the Journal of Mathematical Physics) AD A025 441
- PP 155
Squires, Michael L., "Soviet Foreign Policy and Third World Nations," 26 pp., Jun 1976 (Prepared for presentation at the Midwest Political Science Association meetings, Apr 30, 1976) AD A028 368
- PP 156
Stallings, William, "Approaches to Chinese Character Recognition," 12 pp., Jun 1976 (Reprinted from Pattern Recognition (Pergamon Press), Vol. 8, pp. 87-98, 1976) AD A028 692
- PP 157
Morgan, William F., "Unemployment and the Pentagon Budget: Is there Anything in the Empty Pork Barrel?" 20 pp., Aug 1976 AD A030 455
- PP 158
Haskell, LCdr. Richard D. (USN), "Experimental Validation of Probability Predictions," 25 pp., Aug 1976 (Presented at the Military Operations Research Society Meeting, Fall 1976) AD A030 458
- PP 159
McConnell, James M., "The Gorskov Articles. The New Gorskov Book and Their Relation to Policy," 93 pp., Jul 1976 (To be printed in Soviet Naval Influence: Domestic and Foreign Dimensions, ed. by M. McGwire and J. McDonnell; New York: Praeger) AD A029 227
- PP 160
Wilson, Desmond P., Jr., "The U.S. Sixth Fleet and the Conventional Defense of Europe," 50 pp., Sep 1976 (Submitted for publication in Adelpu Papers, I.I.S.S., London) AD A030 457
- PP 161
Melich, Michael E. and Peet, Vice Adm. Ray (USN, Retired), "Fleet Commanders: Afloat or Ashore?" 9 pp., Aug 1976 (Reprinted from U.S. Naval Institute Proceedings, Jun 1976) AD A030 456
- PP 162
Friedheim, Robert L., "Parliamentary Diplomacy," 106 pp., Sep 1976 AD A033 306
- PP 163
Lockman, Robert F., "A Model for Predicting Recruit Losses," 9 pp., Sep 1976 (Presented at the 84th annual convention of the American Psychological Association, Washington, D.C., 4 Sep 1976) AD A030 459
- PP 164
Mahoney, Robert B., Jr., "An Assessment of Public and Elite Perceptions in France, The United Kingdom, and the Federal Republic of Germany, 31 pp., Feb 1977 (Presented at Conference "Perception of the U.S. - Soviet Balance and the Political Uses of Military Power" sponsored by Director, Advanced Research Projects Agency, April 1976)
- PP 165
Jondrow, James M., "Effects of Trade Restrictions on Imports of Steel," 67 pp., November 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 166
Feldman, Paul, "Impediments to the Implementation of Desirable Changes in the Regulation of Urban Public Transportation," 12 pp., Oct 1976, AD A033 322
- PP 166 - Revised
Feldman, Paul, "Why It's Difficult to Change Regulation," Oct 1976
- PP 167
Kleinman, Samuel, "ROTC Service Commitments: a Comment," 4 pp., Nov 1976, (To be published in Public Choice, Vol. XXIV, Fall 1976) AD A033 305
- PP 168
Lockman, Robert F., "Revalidation of CNA Support Personnel Selection Measures," 36 pp., Nov 1976
- PP 169
Jacobson, Louis S., "Earnings Losses of Workers Displaced from Manufacturing Industries," 38 pp., Nov 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 170
Brechling, Frank P., "A Time Series Analysis of Labor Turnover," Nov 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 171
Ralston, James M., "A Diffusion Model for GaP Red LED Degradation," 10 pp., Nov 1976, (Published in Journal of Applied Physics, Vol. 47, pp. 4518-4527, Oct 1976)
- PP 172
Classen, Kathleen P., "Unemployment Insurance and the Length of Unemployment," Dec 1976, (Presented at the University of Rochester Labor Workshop on 16 Nov 1976)
- PP 173
Kleinman, Samuel D., "A Note on Racial Differences in the Added-Worker/Discouraged-Worker Controversy," 2 pp., Dec 1976, (Published in the American Economist, Vol. XX, No. 1, Spring 1976)

CNA Professional Papers — 1973 to Present (Continued)

PP 174

Mahoney, Robert B., Jr., "A Comparison of the Brookings and International Incidents Projects," 12 pp. Feb 1977

PP 175

Levine, Daniel; Stoloff, Peter and Spruill, Nancy, "Public Drug Treatment and Addict Crime," June 1976, (Published in Journal of Legal Studies, Vol. 5, No. 2

PP 176

Felix, Wendi, "Correlates of Retention and Promotion for USNA Graduates," 38 pp., Mar 1977

PP 177

Lockman, Robert F. and Warner, John T., "Predicting Attrition: A Test of Alternative Approaches," 33 pp. Mar 1977. Presented at the OSD/ONR Conference on Enlisted Attrition Xerox International Training Center, Leesburg, Virginia, 4-7 April 1977